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# TEMPERATURE ERRORS COMPENSATION IN CORIOLIS VIBRATORY GYROSCOPES

## Introduction

Vibratory gyroscopes utilizing Coriolis effect were successfully used in vast amount of different applications since micro fabrication techniques made possible to reduce its cost in mass production along with significant reduction in size [1, 2]. At the same time, Coriolis vibratory gyroscopes (CVG) traditionally occupy niche of low accuracy sensors due to the low stability of its performances under influence of the operational environment. One of the major sources of such instabilities is temperature variations that cause changes in all measurement characteristics of CVGs [3, 4]. In this paper we study the effect of temperature variations on the CVG with cylindrical sensitive element, develop empirical model of the temperature influences, identify its parameters, and develop model of angular rate measurement error due to the temperature variations. Later we validate obtained models using experimental data.

## Temperature related zero-rate output problem

Significant temperature related zero-rate output has been observed during experimental tests of CVG. For the temperature profile, shown in Fig. 1, and zero angular rate, CVG output is shown in Fig. 2 (uncompensated).

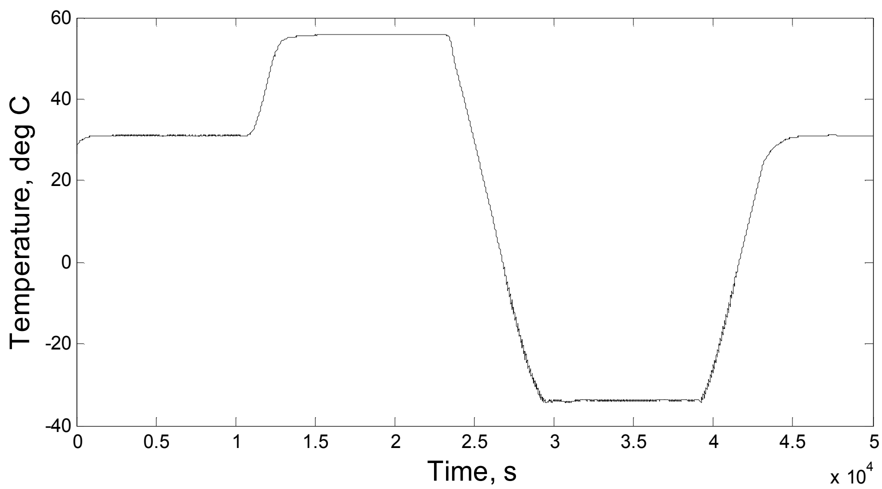


Fig. 1. Temperature profile

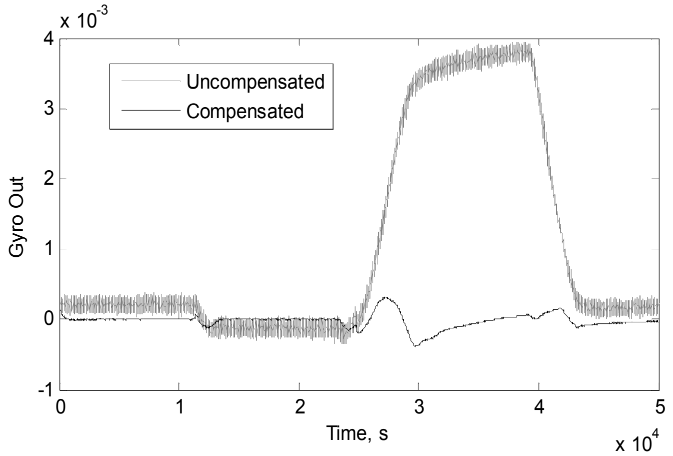

Fig. 2. CVG output with and without temperature compensation

It is believed that temperature variations cause this bias through the temperature dependent cross-damping. In this case excited primary oscillations of the sensitive element will induce secondary (output) oscillations even without external rotation being applied to the sensor.

In order to develop mathematical model for this phenomenon let us first analyse how cross-damping affects dynamics of the CVG sensitive element.

### Sensitive element motion equations

In the most generalized form, motion equations of the CVG sensitive element both with translational and rotational motion could be represented in the following form [5]:

$$\begin{cases} \ddot{x}_1 + 2\zeta_1 k_1 \dot{x}_1 + \left(k_1^2 - d_1\Omega^2\right)x_1 + g_1\Omega\dot{x}_2 + d_3\dot{\Omega}x_2 = q_1(t), \\ \ddot{x}_2 + 2\zeta_2 k_2 \dot{x}_2 + \left(k_2^2 - d_2\Omega^2\right)x_2 - g_2\Omega\dot{x}_1 - \dot{\Omega}x_1 = q_2(t). \end{cases} \tag{1}$$

Here $x_1$ and $x_2$ are the generalized coordinates that describe primary (excited) and secondary (sensed) motions of the sensitive element respectively, $k_1$ and $k_2$ are the corresponding natural frequencies, $\zeta_1$ and $\zeta_2$ are the dimensionless relative damping coefficients, $\Omega$ is the measured angular rate, which is orthogonal to the axes of primary and secondary motions, $q_1$ and $q_2$ are the generalized accelerations due to the external forces acting on the sensitive element. The remaining dimensionless coefficients are different for the sensitive elements exploiting either translational or rotational motion. For the vibrating cylinder sensitive element, for example, $d_1 = d_2 = 1$, $d_3 = 1$, $g_1 = 2$, $g_2 = 2$. For other sensitive elements designs expressions for these coefficients can be found in [5].

If cross damping is present in the system, the motion equations (1) are transformed to the following form

$$
\begin{cases}
\ddot{x}_1 + 2\zeta_1 k_1 \dot{x}_1 + \left(k_1^2 - d_1\Omega^2\right)x_1 + \left(g_1\Omega + 2\zeta_{21}k_2\right)\dot{x}_2 + d_3\dot{\Omega}x_2 = q_1(t), \\
\ddot{x}_2 + 2\zeta_2 k_2 \dot{x}_2 + \left(k_2^2 - d_2\Omega^2\right)x_2 - \left(g_2\Omega + 2\zeta_{12}k_1\right)\dot{x}_1 - \dot{\Omega}x_1 = q_2(t).
\end{cases} \tag{2}
$$

Here $\zeta_{12}$ and $\zeta_{21}$ are the relative cross-damping coefficients. Constant cross-coupling through the damping can be removed by calibration. However, calibration is unable to deal effectively with the varying in time damping due to the temperature variations.

Let us now analyse cross-damping related components in the amplitude of the secondary oscillations. Transforming equations (2) using amplitude-phase complex variables similarly to what has been demonstrated in [6], the following first-order equation for the slowly varying amplitudes ($\ddot{A}_2 \approx 0$) can be produced:

$$
2\left(\zeta_2 k_2 + j\omega\right)\dot{A}_2 + \left(k_2^2 - \omega^2 + 2j\omega k_2\zeta_2\right)A_2 = \left(j\omega g_2\Omega + 2j\omega\zeta_{12}k_1 + \dot{\Omega}\right)A_1. \tag{3}
$$

Here $A_1$ is the constant (does not depend on time) complex amplitude of the primary oscillations

$$
A_1 = \frac{q_{10}}{k_1^2 - \omega^2 + 2jk_1\zeta_1\omega},
$$

and complex amplitudes $A_i$ are expressed in terms of the real amplitudes and phases as $A_i(t) = A_{i0}(t)e^{j\phi_{i0}(t)}$, where $i$ equals 1 or 2 for the primary or secondary oscillations correspondingly. One should also note that disturbances in primary oscillations caused by secondary are considered negligible comparing to the forces from the excitation system.

Applying Laplace transformation to both sides of the equation (3), and solving obtained algebraic equation for the secondary amplitude, results in

$$
A_2(s) = \frac{A_1\left[\left(s + jg_2\omega\right)\Omega(s) + 2j\omega k_2\zeta_{12}(s)\right]}{k_2^2 - \omega^2 + 2j\omega k_2\zeta_2 + 2s\left(k_2\zeta_2 + j\omega\right)}. \tag{4}
$$

Solution (4) can be represented as a sum of the following two components:

$$
\begin{aligned}
A_2(s) &= A_2^{\Omega}(s) + A_2^{\zeta}(s), \\
A_2^{\Omega}(s) &= \frac{A_1\left(j\omega g_2 + s\right)}{\left(s + j\omega\right)^2 + 2\zeta_2 k_2\left(s + j\omega\right) + k_2^2}\Omega(s), \\
A_2^{\zeta}(s) &= \frac{2A_1 j\omega k_2}{\left(s + j\omega\right)^2 + 2\zeta_2 k_2\left(s + j\omega\right) + k_2^2}\zeta_{12}(s).
\end{aligned} \tag{5}
$$

Here $A_2^{\Omega}(s)$ is part of the secondary amplitude due to the input angular rate, and $A_2^{\zeta}(s)$ is due to the cross-damping. Corresponding to (5) transfer functions are hence defined as

$$A_2(s) = W_2^{\Omega}(s)\cdot\Omega(s) + W_2^{\zeta}(s)\cdot\zeta_{12}(s),$$

$$W_2^{\Omega}(s) = \frac{q_{10}\left(j\omega g_2 + s\right)}{\left[\left(s+j\omega\right)^2 + 2\zeta_2 k_2\left(s+j\omega\right) + k_2^2\right]\left(k_1^2 - \omega^2 + 2jk_1\zeta_1\omega\right)}, \tag{6}$$

$$W_2^{\zeta}(s) = \frac{2j\omega k_2}{\left[\left(s+j\omega\right)^2 + 2\zeta_2 k_2\left(s+j\omega\right) + k_2^2\right]\left(k_1^2 - \omega^2 + 2jk_1\zeta_1\omega\right)}.$$

It is important to remember that part of the secondary amplitude due to the cross-damping will be undistinguishable from the one caused by the angular rate. Let us therefore derive transfer function relating input cross-damping to the output angular rate as

$$\Omega^{\zeta}(s) = W_{\Omega}^{\zeta}(s)\cdot\zeta_{12}(s), \tag{7}$$

where $\Omega^{\zeta}(s)$ is the measured erroneous angular rate caused by the cross-damping. Quite apparently unknown transfer function $W_{\Omega}^{\zeta}(s)$ can be expressed using transfer functions from (6) as

$$W_{\Omega}^{\zeta}(s) = \frac{W_2^{\zeta}(s)}{W_2^{\Omega}(s\to 0)} = \frac{2k_2\left(k_2^2 - \omega^2 + 2jk_2\omega\zeta_2\right)}{g_2\left(k_2^2 - \omega^2 + 2k_2\zeta_2 s + 2j\omega\left(s + k_2\zeta_2\right)\right)}. \tag{8}$$

Transfer function (8) can be further simplified using assumptions that are relevant to CVGs with cylindrical sensitive element, and are good approximations for other sensitive elements designs (see [6]). Namely, we can assume that natural frequencies are equal ($k_1 = k_2 = k$) as well as relative damping coefficients ($\zeta_1 = \zeta_2 = \zeta$), and primary oscillations excitation frequency is $\omega = k\sqrt{1-2\zeta^2}$. With these assumptions transfer function (8) becomes

$$W_{\Omega}^{\zeta}(s) = \frac{2k^2\zeta}{g_2\left(s + k\zeta\right)}. \tag{9}$$

Transfer function (9) allows efficient analysis of errors due to the cross-damping, which not only is present in the system, but can vary due to the different reasons.

## Empirical modelling of cross-damping

Assuming that the cross-damping coefficient is a function of the temperature shift $T$ from the calibration temperature, it can be approximated using polynomial as

$$\zeta_{12} = \zeta_{12}(T) \approx \sum_{i=0}^{n} \zeta_i^T T^i \, . \tag{10}$$

Temperature related coefficients $\zeta_i^T$ can be determined experimentally when ambient temperature is known (measured) and angular rate is absent (see Figures 1 and 2). However, in most of the cases we observe angular rate as the gyro output. In order to relate angular rate to the input cross damping, let us use steady state of the transfer function (9) as

$$\Omega(T) = W_\Omega^\zeta(s \to 0)\zeta_{12}(T) \approx \frac{2k}{g_2}\sum_{i=0}^{n} \zeta_i^T T^i = \sum_{i=0}^{n} \Omega_i^T T^i \, . \tag{11}$$

Parameters $\Omega_i^T$ of the cross-damping model (11) can now be identified from the experimental data and found to have the following values: $\Omega_0^T = 1.0792 \cdot 10^{-3}$, $\Omega_1^T = -4.631 \cdot 10^{-5}$, $\Omega_2^T = 7.7044 \cdot 10^{-7}$, $\Omega_3^T = -5.8598 \cdot 10^{-9}$. Influence of the higher order components found to be negligible. In order to validate cross-damping model (11), obtained temperature related angular rate can be subtracted from the gyroscope output, producing compensated output as shown in Fig. 2 (compensated line). As one can see, model (11) successfully compensates bias for the steady temperature, while performs only fair during temperature transitions.

## Temperature compensation system

In order to deal successfully with transient processes in CVG dynamics due to the temperature, let us synthesise temperature compensation system using cross-coupling compensation technique described in [7]. Structure of a simple partial decoupling system is shown in Fig. 3.

Here transfer functions $W_1(s)$, $W_2(s)$, $C_1(s)$, and $C_2(s)$ define dynamics of the CVG sensitive element with respect to cross-damping as

$$\begin{aligned} W_1(s) &= \frac{1}{s^2 + 2\zeta_1 k_1 s + k_1^2}, \\ W_2(s) &= \frac{1}{s^2 + 2\zeta_2 k_2 s + k_2^2}, \\ C_1(s) &= (g_1\Omega + 2\zeta_{21}k_2)s \, , \\ C_2(s) &= (g_2\Omega + 2\zeta_{12}k_1)s \, , \end{aligned} \tag{12}$$

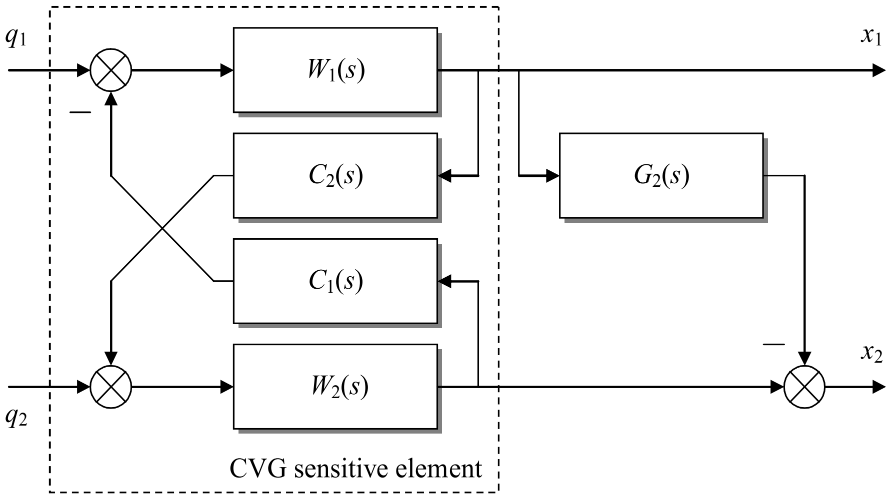


Fig. 3. CVG with the partial decoupling system

and transfer function $G_2(s)$ represents decoupling system and is given by

$$G_2(s) = \frac{2\zeta_{12} k_1 s}{s^2 + 2\zeta_2 k_2 s + k_2^2}. \tag{13}$$

Here $\zeta_{12}$ is the temperature dependent cross-damping coefficient given by (10). By taking temperature measurements from the temperature sensor one can now combine these readings with the measured primary oscillations to implement low level (before demodulation) temperature compensation as shown below in Figure 4.

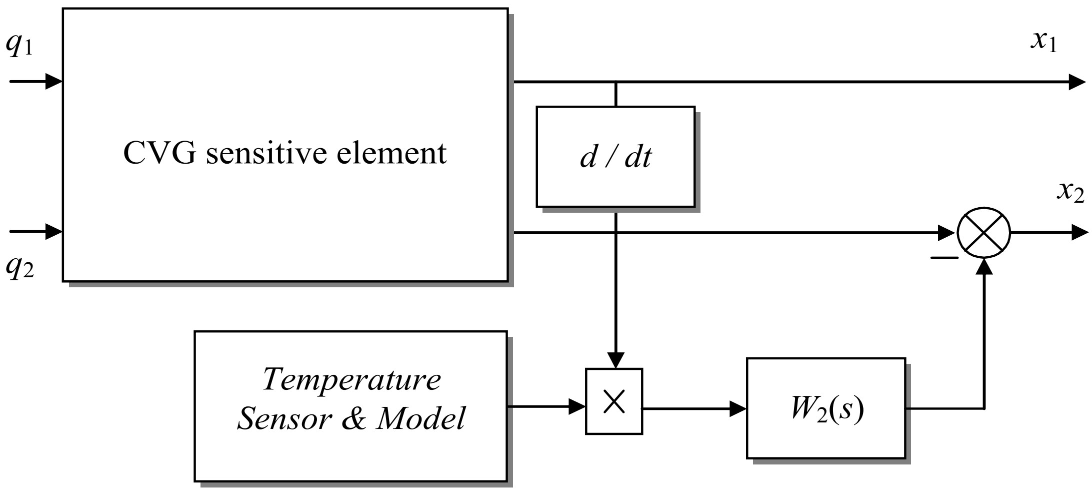


Fig. 4. Low-level temperature compensation system

Results of realistic numerical simulations of this system operation are shown in Figure 5.

In these numerical simulations temperature has sinusoidal shape ranging from – 50 to 50 degrees Celsius and period of 1 s. One can see, that proposed temperature compensation system successfully removed effect of cross-damping variations due to the temperature.

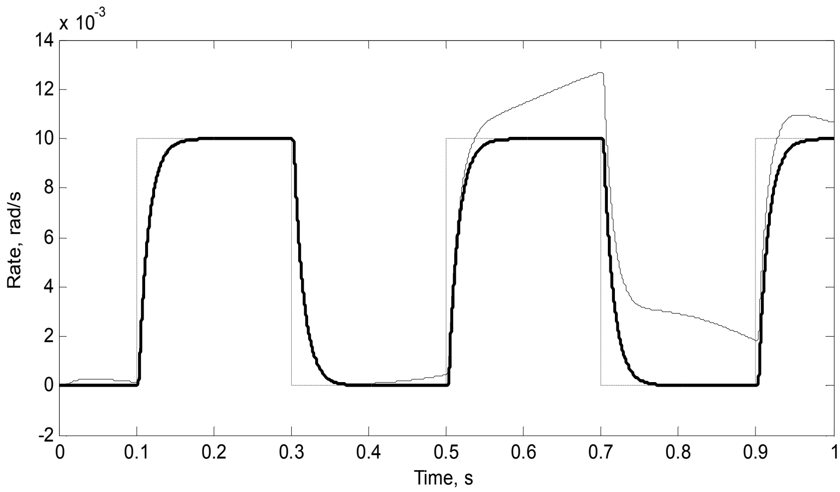


Fig. 5. Temperature compensation system simulation
(dashed line – input angular rate, thin line – uncompensated output, thick line – compensated output)

**Resume**

Developed in this paper model of temperature related errors in CVGs along with the empirical model of the cross-damping have been used to develop low-level temperature error compensation system, which significantly improved undesired influence of the temperature dependent cross-damping. However, proposed system still requires temperature sensor being used in the system. In our future research we plan to use model of cross-damping errors to develop stochastic system of temperature errors compensation that will not require temperature measurements.

**References**

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